Estimating Radar Velocity using Direction of Arrival Measurements

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Abstract

Direction of Arrival (DOA) measurements, as with a monopulse antenna, can be compared against Doppler measurements in a Synthetic Aperture Radar (SAR) image to determine an aircraft's forward velocity as well as its crab angle, to assist the aircraft's navigation as well as improving high-performance SAR image formation and spatial calibration.

Acknowledgements

Contents

Foreword

This report details the results of an academic study. It does not presently exemplify any operational systems with respect to modes, methodologies, or techniques.

Classification

The specific mathematics and algorithms presented herein do not bear any release restrictions or distribution limitations.

This distribution limitations of this report are in accordance with the classification guidance detailed in the memorandum "Classification Guidance Recommendations for Sandia Radar Testbed Research and Development", DRAFT memorandum from Brett Remund (Deputy Director, RF Remote Sensing Systems, Electronic Systems Center) to Randy Bell (US Department of Energy, NA-22), February 23, 2004. Sandia has adopted this guidance where otherwise none has been given.

This report formalizes preexisting informal notes and other documentation on the subject matter herein.

1   Introduction & Background

High-performance Synthetic Aperture Radar (SAR) requires precise knowledge of the relative motion between radar and target scene. This is most often accomplished with Global Positioning System (GPS)-aided Inertial Navigation System (INS). The task of the GPS is to provide absolute references for correcting errors due to noise and drifts in the Inertial Measurement Unit (IMU).

The correction of IMU motion information is termed "alignment" of the IMU. This is often done via an Extended Kalman Filter (EKF) which combines the GPS and IMU data to estimate errors and corrections. Such an algorithm and its implementation is frequently termed the "navigator."[1]

In the absence of GPS-aiding, instrument noise in the INS will cause drifts in the motion data, yielding inaccurate estimates of position, velocity, and angular orientation.

In particular, an error in the velocity estimate will yield an azimuth scaling error in the SAR image, as well as a misfocus or blurring in the image. At non-broadside squint angles, a velocity error will manifest as an unknown and undesired Doppler shift, further manifesting as an additional position shift and illumination error, with attendant deleterious effects on Radar Cross Section (RCS) estimation.

It would be advantageous to be able to accurately estimate the velocity of the radar from the radar data itself. We state for completeness that we require velocity with respect to the ground, and not airspeed.

Most published literature is concerned with geolocation accuracy, particularly of a target. These systems are generally not concerned with estimating all necessary radar velocity parameters with sufficient precision and accuracy to properly form high-performance SAR images.

Paschall and Layne[2] discuss an integrated (GPS, INS, and SAR) targeting system that feeds SAR data back into a Kalman Filter to provide improved target location accuracy. This included monopulse Direction of Arrival (DOA) measurements of specific target pixels. The intent is to improve a target position estimate. However, no specific aircraft or radar velocities are calculated to aid vehicle navigation.

Layne and Blasch[3] report on techniques for improving target location estimates. They use multiple SAR images to refine and improve geolocation of targets by integrating "navigation and SAR measurements in a Kalman filter." Although monopulse measurements are used to determine target direction, aircraft velocity is generally presumed known.

DeBell[4] discusses a combined GPS/INS/monopulse SAR that intends to "reduce the number of channels between the monopulse radar system and the position processor" with a resulting "weight, hardware and cost savings without a corresponding reduction in

performance." He presents a technique to eliminate one axis of a two-axis monopulse system, but presumes that he always retains velocity information from a GPS-aided INS.

Pachter and Porter[5] discuss aiding an INS using "bearings-only measurements of an unknown ground object in three-dimensional space", specifically an "optical flow measurement." They specifically address completely passive techniques to avoid "jamming, spoofing, or interference." Their technique is not suitable for, nor designed for, SAR aiding the INS.

One technique that tries to estimate the line-of-sight velocity for the boresight of an antenna attempts to determine the Doppler centroid of received data. Madsen[6] discusses such a technique. However, such techniques are susceptible to SAR image content, and generally suffer in accuracy and precision. It is well-known that finding centroids is generally less accurate than finding nulls (which tend to be sharper, i.e. more narrow).

Powel[7] presents a "synthetic aperture radar in which radial motion compensation is provided by a monopulse null tracking loop which tracks the null position of a stabilized monopulse antenna, and in which tangential motion compensation is provided by a tangential velocity measurement loop which tracks the cross-over angle of the monopulse antenna pattern." While this will stabilize the Doppler frequency to the boresight of the antenna, it does not address azimuth scaling errors due to the forward velocity of the aircraft.

Moreira[8] presents a method for extracting motion errors using a SAR system. He measures a displacement in azimuth spectra over an interval of time to calculate velocities. His technique essentially requires two or more consecutive synthetic apertures, but falters with scene content that lacks contrast.

One technique that does use radar to determine direction of flight as well as more generally translational motion is embodied in a class of radars called Doppler Navigation radars.[9] An early example is presented by Berger.[10] Such radars make use of multiple distinct beams directed in different bearing directions with respect to the aircraft. The beams are generally with fixed angular differences and non-overlapping. The Doppler information of these beams is compared to determine the true direction of translation of the radar over the ground. These are custom systems that represent additional equipment with additional cost and complexity, as well as size, weight, and power demands on the platform.

We note that a SAR image is essentially a range-Doppler map of the target scene, where azimuth position in the image depends on the pixel content's actual line-of-sight velocity, which in turn depends on its instantaneous squint angle and the radar's velocity. An independent measure of the squint angle to the target pixel would allow us to calculate the radar's velocity, thereby facilitating improved SAR image formation. Such an independent measure is achievable using interferometric DOA techniques using a multi-aperture antenna. A Kalman filter is not required for this calculation.

2 Overview & Summary

In a SAR image, radar velocity and DOA combine to yield a measurable Doppler frequency for any particular pixel. Multi-aperture techniques such as monopulse allow us to derive an independent measure of DOA to each pixel based on interferometric analysis. This independent measure of DOA allows us to decouple the velocity from the DOA in the range-Doppler map with high accuracy and precision, thereby allowing us to calculate a radar velocity.

Any monopulse SAR system (or equivalent) might use this technique. No dedicated distinct non-overlapping antenna beams are required.

Herewith, the radar can assist in aligning the INS, and to keep it aligned, even for large velocity errors, without the need for additional velocity-measuring instrumentation.

"It is no use saying, 'We are doing our best.'
You have got to succeed in doing what is necessary."
-- Winston Churchill

3 Detailed Discussion

The line-of-sight velocity of a radar towards a stationary target is calculated as

$$v_{los} = v_a \cos \theta_s \tag{1}$$

where

v_a = forward velocity of the aircraft, and

θ_s = squint angle between velocity vector and line-of-sight to target. $\qquad$ (2)

We shall presume that the aircraft carrying the radar is flying straight and level.

The Doppler frequency that such a target imparts to a radar echo is calculated as

$$f_d = \frac{2}{\lambda} v_{los} \tag{3}$$

where

λ = nominal wavelength of the radar. $\qquad$ (4)

Note that with this convention, v_{los} is a closing velocity, and positive v_{los} yields a positive Doppler frequency f_d.

Combining these yields

$$f_d = \frac{2}{\lambda} v_a \cos \theta_s. \tag{5}$$

A suitable target would be a particular pixel in a range-Doppler map. For such a pixel, its Doppler frequency f_d could be fairly precisely known. In addition, an independent measure of squint angle θ_s for that pixel can be made with a multiple-aperture antenna using DOA techniques.

With the additional knowledge of λ, then we could solve for aircraft velocity v_a.

However, absolute knowledge of θ_s with sufficient precision and accuracy may often be problematic. Even f_d may sometimes be ambiguous due to insufficient knowledge of antenna orientation with respect to the velocity vector. This leads us to propose a differential approach.

We begin by expanding the squint angle into a reference squint angle and an offset, namely

$$\theta_s = \theta_{s,0} + \phi, \tag{6}$$

where

$\theta_{s,0}$ = a reference squint angle, and

ϕ = offset angle for target pixel. (7)

This lets us expand

$$f_d = \frac{2}{\lambda} v_a \cos\left(\theta_{s,0} + \phi\right). \tag{8}$$

We now make some observations about the nature of these various angles.

- A typical antenna for SAR will often exhibit a fairly small azimuth beamwidth, perhaps on the order of low-single-digit degrees.

- In a single range-Doppler map, we expect the absolute squint angles to the various pixels to be fairly close together; within one antenna azimuth beamwidth of each other.

- We will choose $\theta_{s,0}$ to be the squint angle to the antenna boresight.

- We will normally attempt to operate with the antenna pointed to broadside with respect to the velocity vector, or as near to broadside as possible; broadside being when $\theta_{s,0} = 90$ degrees.

- Offset angles ϕ will typically have magnitude less than one-half the antenna azimuth beamwidth.

These observations allow us to justifiably linearize the cosine with a first-order Taylor series expansion, that is

$$\cos\left(\theta_{s,0} + \phi\right) \approx \cos\left(\theta_{s,0}\right) - \sin\left(\theta_{s,0}\right)\phi. \tag{9}$$

Our expression for Doppler frequency can then be rewritten as

$$f_d = -\frac{2}{\lambda} v_a \sin\left(\theta_{s,0}\right)\phi + \frac{2}{\lambda} v_a \cos\left(\theta_{s,0}\right). \tag{10}$$

This is the equation of a line, where

$$f_d = m\phi + b, \tag{11}$$

where

$$m = \frac{df_d}{d\phi} = -\frac{2}{\lambda} v_a \sin \theta_{s,0} = \text{slope, and}$$

$$b = f_d \big|_{\phi=0} = \frac{2}{\lambda} v_a \cos \theta_{s,0} = \text{y-intercept} \ (f_d\text{-intercept}). \tag{12}$$

In a range-Doppler map, coupled to multi-aperture DOA techniques, each pixel yields a data pair $\left(\phi_i, f_{d,i} \right)$ where the subscript i denotes a particular pair from a particular pixel. The task is to find parameters m and b to best fit this data. It is advantageous to limit our data set to pixels that exhibit at least some minimum Signal-to-Noise Ratio (SNR).

Given a set of pixels and corresponding measures, any of a number of techniques might be employed to fit a line to the data. The technique we will use here attempts to calculate the Minimum Mean Squared Error (MMSE) solution as follows.

First we define the following vectors

$$\underline{\phi} = \left[\phi_1, \phi_2, ..., \phi_I \right]^T,$$

$$\underline{f_d} = \left[f_{d,1}, f_{d,2}, ..., f_{d,I} \right]^T, \text{ and}$$

$$\underline{1} = \left[1,1,...,1 \right]^T. \tag{13}$$

All vectors are of length I, where I is the number of pixels used. The superscript T denotes transpose. We now define the matrix and vector

$$\mathbf{A} = \left[\underline{\phi}, \underline{1} \right], \text{ and}$$

$$\mathbf{b} = \underline{f_d}, \tag{14}$$

which allows us to write the matrix equation

$$\mathbf{A} \begin{bmatrix} m \\ b \end{bmatrix} = \mathbf{b}. \tag{15}$$

We can then solve for the slope and intercept as

$$\begin{bmatrix} m \\ b \end{bmatrix} = \left(\mathbf{A}^T \mathbf{A} \right)^{-1} \mathbf{A}^T \mathbf{b}. \tag{16}$$

We now have parameters for a MMSE best-fit line to the relationship of f_d to ϕ. Furthermore, we may calculate the best-fit actual reference squint angle as

$$\cot \theta_{s,0} = -\frac{b}{m} = \text{x-intercept} \ (\phi\text{-intercept}). \tag{17}$$

Recall that in the neighborhood of $\theta_{s,0} = 90$ degrees, this is a small number. These quantities are illustrated in Figure 1.

We may calculate the sine of the squint angle as

$$\sin\theta_{s,0} = \frac{-m}{\sqrt{m^2 + b^2}} \; . \tag{18}$$

This allows us to calculate the aircraft velocity as

$$v_a = \left(\frac{\lambda}{2}\right)\sqrt{m^2 + b^2} \; . \tag{19}$$

The line-of-sight velocity in the direction of the antenna beam boresight is

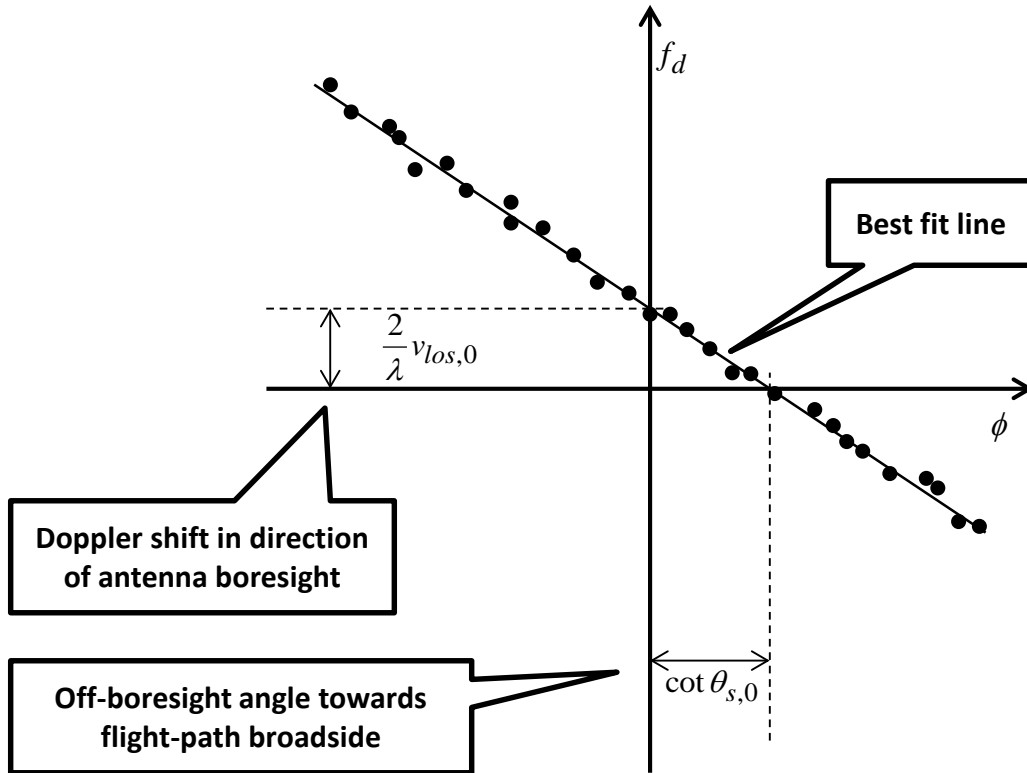$$v_{los,0} = \left(\frac{\lambda}{2}\right)b \; . \tag{20}$$

Figure 1. Relationship of Doppler to off-boresight angle.

Both the aircraft velocity v_a, and the squint angle $\theta_{s,0}$ have thus been calculated. These quantities may now be used for any of the following tasks.

- Determining the forward ground-speed of the aircraft.

- Determining the crab-angle of the aircraft, via the difference between $\theta_{s,0}$ and the actual gimbal to aircraft body angle.

- Determining the line-of-sight velocity of the radar in the direction of the antenna boresight.

- Providing correct azimuthal scaling of the SAR mage.

- Allowing correct application of antenna beam-pattern corrections to radiometrically calibrate the SAR image.

For many situations, a single-axis azimuth monopulse measure (or equivalent) will suffice. This is true when either the azimuth monopulse axis is aligned with the platform velocity vector, or when the target scene is flat (which is reasonably typical for many SAR images). However, for a target scene with significant topographical relief, and a large line-of-sight velocity, an additional elevation DOA measure could refine the relationship of DOA to pixel Doppler measure. This additional elevation DOA measure might come from an additional elevation monopulse antenna characteristic, or perhaps from a Digital Terrain Elevation Data (DTED) database.

Concept of Operations

We present a block diagram in Figure 2 that outlines the steps in collecting and processing radar data to determine the desired velocity components.

We begin with selecting an initial velocity estimate. This might be a best existing estimate from the navigator, the current aircraft measured airspeed, a best guess at the aircraft's airspeed, or even zero.

Next we select an imaging geometry for our measurements. This should be near to broadside of the aircraft's flight direction, or it might be broadside to the aircraft's body, with a range and depression angle that is expected to land on the ground.

Then we collect coincident synthetic apertures of radar data for each phase center or equivalent. This might be sum and difference channel data for a monopulse antenna.

All data is then processed into SAR images, one for each channel of data.

Pixels are then selected in the images that meet some minimum SNR (perhaps 20 dB).

For each selected pixel, its Doppler frequency is recorded against its calculated angle off boresight.

Next we calculate a best-fit line to the Doppler vs. angle data.

From the line, we then extract and/or calculate parameters of interest, including aircraft forward velocity, velocity in the direction of antenna boresight, and off-boresight angle.

These parameters are then used to update the navigator for the radar and/or aircraft.

During the data collection, for maximum accuracy and precision, we desire to keep the antenna pointed to a single Scene Reference Point (SRP) on the ground, in a spotlight fashion. However, to do so requires knowledge of aircraft velocity, which is of course what we desire to determine. A wrong velocity estimate will degrade the parameter estimates, but not so much that they will be rendered useless. Consequently we may iterate this procedure to continuously refine the velocity estimates to first optimally determine them with maximum accuracy and precision, and then to track any changes.

In addition, from one iteration to the next we may alter the imaging geometry if we wish.

Furthermore, if we anticipate a poor initial velocity estimate, we may begin with a relatively coarse resolution SAR image.

It is anticipated that this process may run continuously during a stripmap SAR imaging operation.

```
┌─────────────────────────┐
│   Select initial aircraft│
│   velocity estimate      │
└─────────────────────────┘
            │
            ▼
┌─────────────────────────┐
│   Select suitable imaging│
│   geometry               │
└─────────────────────────┘
            │
            ▼
┌─────────────────────────┐
│   Collect multi-aperture │
│   SAR data               │
└─────────────────────────┘
            │
            ▼
┌─────────────────────────┐
│   Process all data into  │
│   respective images      │
└─────────────────────────┘
            │
            ▼
┌──────────────────┐   ┌─────────────────────────┐
│ Define minimum SNR│──▶│   Select pixels for      │
│ for image pixels  │   │   DOA/Doppler analysis   │
└──────────────────┘   └─────────────────────────┘
            │
            ▼
┌─────────────────────────┐
│   Plot Doppler versus off-│
│   boresight angle        │
└─────────────────────────┘
            │
            ▼
┌─────────────────────────┐
│   Fit line to measured data│
└─────────────────────────┘
            │
            ▼
┌─────────────────────────┐
│   Extract new velocity   │
│   component estimates    │
└─────────────────────────┘
            │
            ▼
┌─────────────────────────┐
│   Update navigator       │
└─────────────────────────┘
```
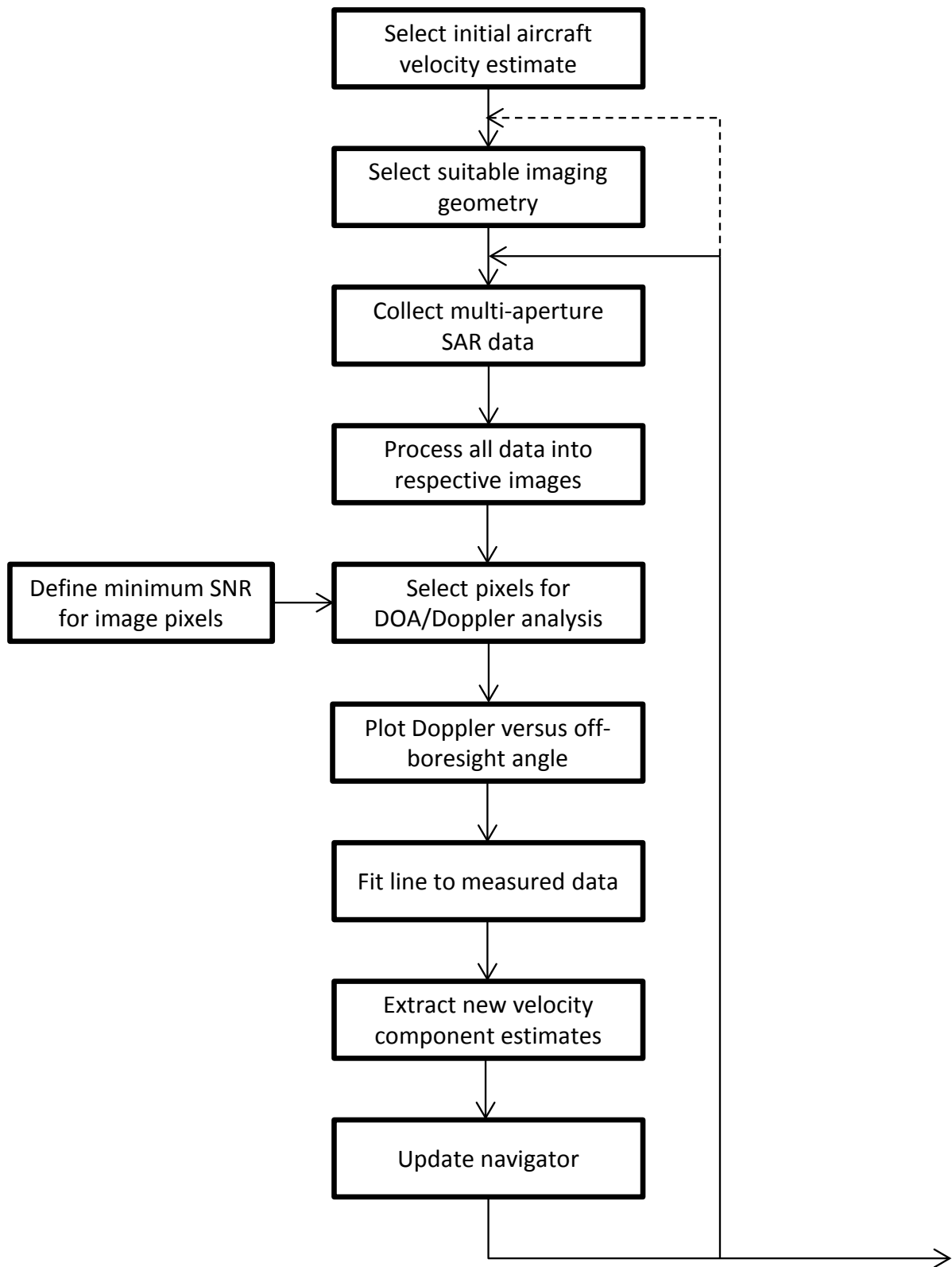
Figure 2. Processing outline.

"The secret of getting ahead is getting started."
-- Mark Twain

4 Conclusions

In this report, we describe a technique for determining velocity parameters using DOA measures. The essence of this technique is

- Two or more independent DOA measures can be compared to their Doppler measures to determine a general relation between the two.

- A line might be fit to this data.

- The line's parameters (slope and intercepts) can be used to calculate aircraft forward velocity, and velocity in the direction of the boresight of the antenna.

- This velocity data can be fed back to the radar's and/or the aircraft's navigator to improve their performance.

"Efforts and courage are not enough without purpose and direction."
-- John F. Kennedy

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